Review of Target Tracking Algorithms Based on Bayes Filtering

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Abstract: Target tracking technology is widely used in the military, traffic, medical, and other fields. Target tracking can be divided into single target tracking and multi-target tracking according to the number of tracks. This paper reviews the research on common algorithms involved in target tracking from the above two aspects. One of the cores of the research is to effectively filter the single target tracking by measurement, and accurately estimate the state of the target in real-time. This paper introduces the related algorithms based on Bayesian filtering that can solve this problem. For example, the Kalman filter is applied to a linear system, the extended Kalman filter is applied to the nonlinear system, and the particle filter. The core of this paper also includes the most important steps in multi-target tracking to solve the uncertain factors in observation samples and complete the correlation between observation samples and trajectory sets. Four algorithms are introduced to solve data correlation problems, such as the global nearest neighbor, joint probability data association, multivariate hypothesis tracking, probability hypothesis density filtering, etc. Finally, this paper summarizes various algorithms to solve target tracking in different situations, discusses their advantages and disadvantages, and puts forward the future problems of data association research.

1. Introduction

Target tracking is an important problem in the computer field and is widely used in intelligent video surveillance, automatic detection, robot visual navigation, and other fields [1]. It is a method to estimate the current state of single or multiple targets by propagating the belief over the state space over time using sensor measurements [2]**.** The state mainly describes the position, velocity, and additional motion components of the target [2]. The key links in the process of target tracking generally include target detection, target location estimation, target model building, and state filtering prediction. Firstly, the existence of the target is detected by a sensor, and observation information is generally obtained by sampling sensing signals such as light, sound, images, or videos [3]. Secondly, the observation information is correlated with the target state to use mathematics to calculate the position of the target, and finally, determine the position of the target in each step to complete the tracking process [3]. This paper will summarize the commonly used algorithms for single target tracking and multi-target tracking respectively.

Bayes filter generally refers to a kind of filtering technology based on Bayes theorem, which recursively calculates the interesting quantities such as the state's poster probability distribution, priority probability distribution, state estimation and state prediction, etc [4]. Bayes filtering originated from the research of filtering problems in linear dynamic systems. Among them, the Kalman filter in the 1960s obtained the state estimation and the state covariance matrix by calculating the Kalman gain, and then a series of improved algorithms appeared one after another [4]. Although under the minimum mean square error criterion, when the state equation and observation equation in the state-space model of a linear dynamic system are linear equations and the process noise and observation noise are Gaussian white noise, the state estimation obtained by KF is proved to be equal to Bayes optimal estimation [4]. In addition, the research on Bayes filter in nonlinear dynamic systems has also made rapid development. The Extended Kalman filter is the first widely recognized nonlinear Bayesian filtering algorithm [5]. Its basic idea is to truncate the Taylor series expansion of the state equation or observation equation near the state point in the state-space model in the first order to linearize the nonlinear equation and use Kalman gain to calculate the state estimate and state covariance [4]. Unlike the above-mentioned nonlinear Bayesian filtering algorithm, the filtering algorithm is based on sequential Monte Carlo technology, which is called Sampling Importance resampling (SIR) [6]. The process of SIR is divided into important sampling and resampling. It approaches the theoretical posterior probability distribution of the state by recursively estimating the empirical distribution composed of a group of weighted random examples. Many similar algorithms have been developed based on SIR, and these related improved algorithms are now collectively referred to as particle filters (PF) [4].

The basic idea of multi-target tracking was first put forward in 1955, which laid a solid foundation for the future development of multi-target tracking [7]. In 1964, Sittler scholars proposed a multi-target tracking method based on data association by analysing the data association in the process of multi-target tracking [7]. In the following decades, scholars such as BAR-SHALOM and singer put forward many classical methods of multi-target tracking based on data association [8]. Data association is an important stage in the process of multi-target tracking, and its essence is to find the optimal associated target pair based on target detection. The association algorithms reviewed in this paper include Global Nearest Neighbour, Joint Probabilistic Data Association, Multiple Hypothesis Tracking, and Probabilistic Hypothesis Density Filter [9]. Because the effective echo may all originate from the target with different probability and considering that the common echo does not originate from only one target, it may belong to different targets. GNN algorithm is based on the nearest neighbour algorithm, considering the possibility of correlation between measured values and multiple targets, that is, the observation point closest to the target in this state vector estimation does not necessarily come from the target [10]. BAR-SHALOM et al. proposed a data association algorithm for multi-target tracking, that is, Joint Probabilistic Data Association (JPDA) [8]. The purpose of the JPDA algorithm is to calculate the probability that each measurement is associated with various possible source targets (or trajectories) [11]. Multiple hypothesis tracking (MHT) is a tracking algorithm for infrared imaging targets, which is based on the all-neighbour optimal filter and the aggregation probability proposed by BAR-SHALOM [12]. The algorithm correlates the data according to the measurement data of multiple scanning cycles, including current measurement and historical measurement [13]. Probability hypothesis density filtering (PHD) is proposed by Mahler. It is different from the general random set method. It is an algorithm for the probability hypothesis density recursive operation [7].

Many scholars have studied the target tracking algorithm and proposed many classic algorithms, but the multi-target tracking data association algorithm needs further research. We can propose four research directions: (1) multi-target algorithm combined with deep learning; (2) Feature extraction and multi-type feature measurement methods; (3) Selecting the appropriate association algorithm according to specific scenarios is a problem that needs further research in practical application.

2. Bayes Filter

According to the obtained observation, at every time k, the posterior probability density function of the target state vector x_i at time k is finally obtained by using the target state vector x_{k-1} at time $k-1$ and all the measured values Z_u from the initial time to time k. Bayes filtering algorithm includes a prediction step and an update step. In the prediction step, the prior probability density function of the state vector is predicted by using the system model [14]. According to the Bayes filtering principle, it is assumed that the prior probability density of the initial state x_0 of the target is known and can be expressed as $p_0(x_0|Z_0)$. When the sensor has not obtained the observed value z_k at time, it is necessary to deduce the posterior probability density $p_{k-1}(x_{k-1}|Z_{1:k-1})$ at time $k-1$ to the prior probability density $p_{k|k-1}(x_k|Z_{1:k-1})$at time k, which is defined as

$$p_{k|k-1}(x_k|Z_{1:k-1}) = \int f_{k|k-1}(x_k|x_{k-1})p_{k-1}(x_{k-1}|Z_{1:k-1})dx_{k-1}. \quad (1)$$

The update step is the process of correcting the prior probability density function by using the latest measurement value, and then obtaining the posterior probability density function [14]. Using the latest measured value z_k of the sensor and the prior probability density $p_{k|k-1}(x_k|Z_{1:k-1})$ at k time, the posterior probability density at k time is

$$p_{k|k-1}(x_k|Z_{1:k}) = \frac{g_k(Z_k|x_k)p_{k|k-1}(x_k|Z_{1:k-1})}{\int g_k(Z_k|x_k)p_{k|k-1}(x_k|Z_{1:k-1})dx_k}. \tag{2}$$

Equation 2 constitutes the recursive formula of Bayes filter. In the process of target estimation, there are two measures to evaluate the obtained state estimation, namely expected a poster and maximum a poster, which can be defined as

$$\hat{X}_k^{EAP} = \int x_k p_k(x_k|Z_{1:k})dx_k, \tag{3}$$

And

$$\hat{X}_k^{MAP} = arg\,sup\,p_k(x_k|Z_{1:k}). \tag{4}$$

It can be seen from Equations 2 and 3 that there is an integral operation in the Bayes recursion process, which makes it difficult to use the Bayes filter in practical engineering applications. Approximation techniques are developed to implement Bayes filter, with various representations of state posteriors. The following representative filter algorithms are linear Bayes filter, particle filter, extended Kalman filter, sigma point Kalman filter, and so on.

3. Target State Estimation

Single target tracking uses a certain model to estimate the possible position of the target in the future [15], and the representative methods include Kalman filter, extended Kalman filter, particle filter, and so on.

3.1 Kalman Filter

Kalman filter is a commonly used state estimation algorithm for linear system state prediction [16]. By combining the prior estimation of the system state with the observation data of the next moment, we can roughly estimate the current state of the system, and its essence is still an estimation method based on the Bayes model [17]. First, we assume that the state equation of the linear system and the observation equation of the system are

$$x_k = Ax_{k-1} + Bu_k + R_k, \tag{5}$$

And

$$y_k = Hx_k + Q_k, \tag{6}$$

where x is the state vector; A is the state transition matrix; B is the control matrix; u is the control vector and R_k is the noise system. y is the measured value, H is the transformation matrix from the state variable to the measured variable, and Q_k is the measurement noise [17].

Kalman filter can be divided into two stages: prediction stage and update stage. In the prediction stage, the best estimate of the last moment is used to predict the current state, which can be defined as

$$\widehat{x_k^-} = E(x_k) = E(Ax_{k-1} + Bu_k + R_k), \tag{7}$$

$$\widehat{x_k^-} = A\widehat{x_{k-1}} + Bu_k, \tag{8}$$

And

$$P_k^- = Cov(x_k - \widehat{x_k^-}) = AP_{k-1}A^T + \Sigma_R, \tag{9}$$

Where $\widehat{x_k^-}$ is the state estimate at k; P_k^- is the variance of the state estimate at k; Σ_R is system noise covariance matrix [17].

In the update stage, the predicted value in the prediction stage is corrected by the observed value at the current time, so as to obtain the optimal estimate at the current time,

$$K = P_k^- C^T \left(CP_k^- C^T + \Sigma_Q\right)^{-1}, \tag{10}$$

$$\widehat{x_k} = \widehat{x_k^-} + K(y_k - C\widehat{x_k^-}), \tag{11}$$

$$P_k = (1 - KC)P_k^-. \tag{12}$$

K is Kalman gain; y_k is the measured value at k time; $\widehat{x_k}$ and P_k are the optimal estimation value and the optimal estimation variance at k time respectively. Σ_R is the measurement noise covariance matrix.

3.2 Extended Kalman Filter

Extended Kalman filter (EKF) is a Bayesian filtering algorithm applied to non-linear system [16]. Its basic idea is to expand the nonlinear model by first-order Taylor expansion around state estimation, and then apply the Kalman filtering formula of the linear system. Let the nonlinear state space model be

$$x_k = f(x_{k-1}, u_k) + W_k, \tag{13}$$

And

$$z_k = h(x_k) + V_k. \tag{14}$$

After the extended Kalman filter linearizes the nonlinear function locally are

$$f(x_k, u_k) \approx f(\widehat{x_k}, u_k) + A_k(x_k - \widehat{x_k}), \tag{15}$$

$$h(x_k) \approx h\left(\hat{x}_{k|k-1}\right) + C_k\left(x_k - \hat{x}_{k|k-1}\right), \tag{16}$$

$$A_k = \left|\frac{df}{dx_k}(\widehat{x_k}, u_k)\right|, \tag{17}$$

And

$$C_k = \left|\frac{dh}{dx_k}\left(\hat{x}_{k|k-1}\right)\right|, \tag{18}$$

Then the extended Kalman filter equation can be obtained by applying the Kalman filter equation

$$\hat{x}_{k|k-1} = f\left(\hat{x}_{k-1|k-1}, u_k\right), \tag{19}$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k, \tag{20}$$

$$\tilde{y}_k = z_k - h(\tilde{x}_{k|k-1}), \tag{21}$$

$$S_k = H_k P_{k|k-1} H_k^T + R_k, \tag{22}$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1}, \tag{23}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k y_k, \tag{24}$$

And

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}. \tag{25}$$

$\hat{x}_{k|k-1}$ is the predicted state estimate; $P_{k|k-1}$ is the predicted covariance estimate; $\tilde{y}_k$ is the measured residual value; S_k is the residual covariance; K_k is the optimal Kalman gain; $\hat{x}_{k|k}$ the updated state estimate; $P_{k|k}$ is the updated covariance estimate; $P_{k-1|k-1}$ is the posterior error covariance matrix, F_k is the observation transformation matrix and the Jacobian matrix of the function f(x). H_k is the observation transformation matrix and the Jacobian matrix of the function h(x) [17].

3.3 Particle Filter

Particle filter is different from the above-mentioned nonlinear Bayes filtering algorithm, it is a filtering algorithm based on sequential Monte Carlo technology, which is called Sampling Importance Resampling (SIR) or Bootstrap Filter [15]. The conditional posterior probability density of weighted particle characterization state is defined as

$$P(x_k|z_{1:k}) \approx \sum_{i=1}^{N} w_k^i \delta(x_k - x_k^i). \tag{26}$$

This algorithm consists of two steps: Importance Sampling and Resampling, which approximates the theoretical post-probability distribution of the state by recursively estimating the empirical distribution composed of a group of weighted random particles. Importance Sampling. It refers to updating the particle state based on the estimated state value $x^{(k-1)}$ at $(k-1)$ time and the state transition equation to obtain the prior distribution $p(x_k|z_{1:k})$, and selecting an importance distribution function $q(x_k^i|x_{k-1}^i, z_k)$ as the sub-optimal sampling data. The prior probability density distribution function of the standard particle filter algorithm is

$$q(x_k^i|x_{k-1}^i, z_k) = p(x_k^i|x_{k-1}^i). \tag{27}$$

Then update the particle weight with the latest measurement value, and the weight update formula is

$$\omega_k^i \propto \frac{w_{k-1}^i p(z_k|x_k^i) p(x_k^i|x_{k-1}^i)}{q(x_k^i|x_{k-1}^i, z_k)}, \tag{28}$$

And normalization processing is

$$\omega_k^i = \omega_k^i / \sum_{i=1}^{N_s} \omega_k^i. \tag{29}$$

The x least mean square estimation is

$$\hat{x}_k \approx \sum_{i=1}^{N_s} \omega_k^i \, x_k^i. \tag{30}$$

Because the current measurement value is not considered, when the likelihood function is in a peak state or at the tail of the state transition probability density, a large sampling deviation will occur. With the increase of iteration times, a large number of particle weights are almost zero, and the particle set can't express the actual posterior probability distribution, which is the degradation problem of the particle filter [14]. To avoid degradation, resampling technology is introduced after weight updating. Because SIR resamples the particles according to the weight of each particle in resampling, the purpose is to remove the particles with smaller weight and increase the particles with larger weight. This can make the particles with larger weight be selected many times, and most particles are eliminated because of their smaller weight, thus avoiding the dilemma of particle degradation faced by the early filtering algorithm based on Monte Carlo sampling technology [14]. Due to the advantages of SIR, such as easy realization, easy improvement, and flexible and effective handling of filtering problems in nonlinear non-Gaussian dynamic systems, many similar algorithms have been developed based on SIR, which is collectively referred to as particle filter or Sequential Monte Carlo Methods.

3.4 Discussions

Kalman filter realizes the state estimation of the target system through the iteration of prediction and update stages. In the process of target tracking, there are all kinds of interference noises at the measurement time of the target. Kalman filter is essential to reduce or even eliminate noise based on the motion characteristics of the target to achieve the optimal estimation effect [18], has the following advantages. (1) The system gain can be calculated dynamically, and the same filter design is suitable for multiple computers. (2) It uses covariance matrix to quickly and accurately measure the estimation accuracy. (3) By analyzing the change of residual error, it is judged whether the current target motion model is reliable. (4) The Kalman gain sequence is adaptively changed according to the motion state, to achieve the optimum filtering effect.

4. Multiple Target Data Association

Although the above algorithms can have significant benefits in target tracking, they are used in single target tracking (STT) most time. In the multi-target tracking system, the scene is more complex, and the number and category of tracking targets are often uncertain, so data association is particularly important in the whole tracking system. Different from STT, the goal of the Multiple targets tracking algorithm is to estimate the number and state of targets simultaneously. A variety of algorithms are needed in the process. This paper mainly adopts Global Nearest Neighbour (GNN), Joint Probabilistic Data Association (JPDA), Multiple Hypothesis Tracking (MHT), Probabilistic Hypothesis Density (PHD), etc.

4.1 Global Nearest Neighbour

The Global Nearest Neighbour approach can receive the currently scanned data and cluster the data [19]. The global nearest neighbour method can receive the currently scanned data and cluster the data. The state equation and measurement equation of discrete model k time of typical target tracking system [20] are

$$x(k) = F(k)x(k-1) + \omega(k), \tag{31}$$

And

$$z(k) = H(k)x(k) + v(k). \tag{32}$$

$x(k)$ is the state vector; $F(k)$ is the state transition matrix, and $z(k)$ is the target measurement vector, $H(k)$ is the Jacobi matrix at the estimated value of $x(k)$ state; $\omega(k)$ and $v(k)$ are process noise and measurement noise respectively, and their covariance matrices are Q (k) and R (k) respectively.

In GNN algorithm,

$$\varepsilon_{ij} = \begin{cases} 1 & Observation\ point\ j\ is\ assigned\ to\ track\ i \\ 0 & Observationpointjisn'tassignedtotracki \end{cases}, \tag{33}$$

and suppose that m measurements need to be divided into n targets, such as

$$\begin{cases} \sum_{i=1}^{n} \varepsilon_{ij} <= 1 & Each\ target\ is\ assigned\ a\ maximum\ of\ one\ measurement \\ \sum_{j=1}^{m} \varepsilon_{ij} <= 1 & Each\ measurement\ is\ assigned\ to\ a\ maximum\ of\ one\ target \end{cases}. \tag{34}$$

Suppose the residual of measurement i and target j, denoted as v_{ij}, and equation is

$$v_i(k+1) = z(k+1) - H_i(k+1)x(k+1|k). \tag{35}$$

Covariance matrix of residuals is

$$S_i(k+1) = S_i(k+1|k)H'_i(k+1) + R(k+1). \tag{36}$$

Normalized distance measurement i and target j is

$$d_{ij}^2 = d_{ij}^2 S^{-1} v_{ij}. \tag{37}$$

In the traditional GNN algorithm, the factor of statistical distance is mainly considered, and the equation of cost function is

$$argmin \sum_{i=1}^{n} \sum_{j=1}^{m} \varepsilon_{ij}\, d_{ij}^2. \tag{38}$$

If only the statistical distance is considered as the cost function, there are some limitations: for the same spatial distance, the statistical distance is different when the innovation covariance is different, while the Gaussian probability considers the influence of innovation covariance [20], which is a more reasonable evaluation function, as follow:

$$\wedge_{ij} = \frac{1}{\sqrt{|2\pi S_i|}} exp\left[-\frac{1}{2} d_{ij}^2\right]. \tag{39}$$

Let the confidence factor of the model be $exp(\mu)$, and the corrected cost function is

$$\wedge_{ij}^{*} = \wedge_{ij}\ exp(\mu), \tag{40}$$

And

$$E(k) = argmin \sum_{i=1}^{n} \sum_{j=1}^{m} \varepsilon_{ij} \wedge_{ij}^{*}. \tag{41}$$

GNN algorithm enhances the stability of the relationship and considers various possibilities of data association. The optimal solution is an association set, in which the sum of the possibilities of each association event is the largest.

4.2 Joint Probability Data Association

As the goal of Multiple target tracking is to estimate the number and status of targets at the same time, besides the fact that the number of targets changes randomly with time, the received measured values will also be affected by other uncertain factors. The joint probability data association (JPDA) method can be used to solve the measurement uncertainty in MTT [11]. Cost function is the balance between cardinality estimation performance and target location performance, which can be used to formulate an analysis cost function for MTT [21]. By minimizing the cost function, the confidence level of JPDA can be increased and then improving the estimation performance. JPDA is a mature multiple target data association method, based on the probabilistic reasoning [11]. Firstly, suppose there is a relationship between the goal and the measure, and then associate the measure with the goal. It must be satisfied that each measurement except virtual measurement is assigned to at most one target, and each target is uniquely assigned to one measurement. On the basis of this hypothesis, the equation of joint association hypothesis is $\theta_k = \{\theta_k^i\},\ i \in \{1,2,\ldots\ldots N_{k|k-1} + M_k\}$. For each pre-existed target $i \in \{1,2,\ldots\ldots N_{k|k-1}\}$,and define $\theta_k^i \in \{0,1,\ldots,M_k\}$as the association hypothesis, where $N_{k|k-1}$is defined as the predicted number of targets at scan k. Since we don't have any information about the birth/death of the target when scanning k before receiving the measured value, define $N_{k|k-1} = N_{k-1|k-1}$. The single association event $\theta_k^i = j$ stands for the fact that the jth measurement originates from the ith target and $\theta_k^i = 0$ represents miss detection. Then new track for each measurement $j \in \{1,2,\ldots,M_k\}$at scan k is created, and the association event for these new targets are defined by $\theta_k^{N_{k|k-1}+j} \in \{N_{k|k-1}+1,\ldots,N_{k|k-1}+M_k\}$. If $N_{k|k-1}+j$ is associated with the jth measurement, then $\theta_k^{N_{k|k-1}+j} = N_{k|k-1}+j$. Assuming that each individual correlation event is independent, the minimum mean square error (MMSE) estimation of each target is expressed as

$$p(x_k^i | X_k^i, Z_k) = \sum_{\theta_k^i} p(x_k^i | \theta_k^i, X_k^i, Z_k) p(x_k^i | X_k^i, Z_k). \tag{42}$$

According to Bayes theory, the existence-conditioned marginal association probability $p(\theta_k^i | X_k^i, Z_k)$ is defined as

$$p(\theta_k^i | X_k^i, Z_k) = \frac{p(\theta_k^i, X_k^i | Z_k)}{p(X_k^i | Z_k)} = \frac{p(X_k^i | \theta_k^i, Z_k) p(\theta_k^i | Z_k)}{p(X_k^i | Z_k)}. \tag{43}$$

The hypothesis-conditioned existence probability $p(X_k^i | \theta_k^i, Z_k)$ is given by

$$p(X_k^i | \theta_k^i, Z_k) \propto \begin{cases} \dfrac{p(x_k^i | Z_{k-1})(1-P_D)}{1-p(x_k^i | Z_{k-1}) + p(x_k^i | Z_{k<=-1})(1-P_D)}, & \theta_k^i = 0 \\ 1, & \theta_k^i = j \\ \dfrac{P_D \lambda_B p(z_k^j | x_b)}{\lambda_{FA} + P_D \lambda_B p(z_k^j | x_b)}, & \theta_k^{N_{k-1}+j} = N_{k-1}+j \end{cases} \tag{44}$$

And the posterior existence probability is given by

$$p(X_k^i | Z_k) = \sum_{\theta_k^i} p(\theta_k^i, X_k^i | Z_k), \tag{45}$$

With

$$p\left(\theta_k^i, X_k^i \middle| Z_k\right) = p\left(X_k^i \middle| \theta_k^i, Z_k\right) p(\theta_k | Z_k). \tag{46}$$

According to the law of full probability, it can be theoretically calculated by enumerating all possible joint hypotheses as:

$$p\left(\theta_k^i = j \middle| Z_k\right) = \sum_{\theta_k^i(\in\theta_k)=j} p(\theta_k | Z_k). \tag{47}$$

Where the posterior distribution of the joint association event $p(\theta_k|Z_k)$ is

$$p(\theta_k|Z_k) \propto \left[\prod_{i\in[N_{k|k-1}]\theta_k^i=0} 1 - P_D p\left(x_k^i \middle| Z_{k-1}\right)\right] \times \left[\prod_{i\in[N_{k|k-1}]\theta_k^i=j} P_D p\left(x_k^i \middle| Z_{k-1}\right) p\left(z_k^j \middle| x_{k|k-1}^i\right)\right]$$
$$\times \left[\prod_{\theta_k^{N_{k|k-1}+j}=N_{k|k-1}+j} \lambda_{FA} + P_D\, \lambda_B p\left(z_k^j \middle| x_b\right)\right]. \tag{48}$$

In Equation 48, x_b represents the candidate state of the new target. In a word, every track of JPDA is updated by Eqs. The number of targets can be calculated by the confirmed trajectory. Therefore, JPDA provides a complete framework for MTT and tracking management.

4.3 Multiple Hypothesis Tracking

The implementation of MHT is mainly divided into the following four steps. First, generate the measurement point trace data set, and respectively associate the newly received measurement point trace with the previous assumptions, that is, the data set cluster [13]. The second step is to consider not only the possibility of false alarm, but also the possibility of new targets for each echo, and consider the hypothesis at time k as the result of the correlation between a hypothesis at time k-1 and the current data set. The third step is to eliminate infeasible assumptions [13]. The fourth step is to convert the optimal hypothetical track into a confirmed track. Generating and deleting hypotheses is the core algorithm of MHT [13]. If there is no reasonable strategy, the number of hypotheses generated will increase exponentially with the number of targets, the number of false alarms and the number of data frames processed, and the amount of calculation and storage will be huge [13]. Therefore, effective combination and deleting the number of assumptions of rapid explosion are the key steps of MHT's real-time application in engineering. Therefore, MHT must adopt reasonable strategies to hypothesis generation and hypothesis deletion [13]. In this paper, the k-cycle track I is calculated by using the following formula, The probability of I, $\theta^{k,I}$is calculated as

$$P_r\{\theta^{k,I}|Z^t\} = \frac{1}{c}\frac{v!O!}{m_k!} F(O)N(v)V^{-O-v} \tag{49}$$

$$\times \prod_{i=1}^{m_k}\{N_{t_i}[z_i(k)]\}^{\frac{i}{t}} \prod_t (P_D^t)^{\frac{w}{t}} (1-P_D^t)^{1-w/t} \times P_r\{\theta^{k-1,s}|Z^{k-1}\}, \tag{50}$$

Where C is the normalization constant factor, $\mu_F(\emptyset)$ and $\mu_N(v)$ a priori probability quality function (PMF) of false measurement number ∅ and new target number respectively, P_D^t is the detection probability of track t, $\theta^{k-1,s}$ is $\theta^{k,i}$ historical track.

MHT algorithm is superior in performance, but it needs to exhaust the combination of associated hypotheses and calculate the probability of each hypothesis, which requires a high amount of calculation and storage of the system.

4.4 Probability Hypothesis Density Filter

Probability hypothesis density is the first moment of multi-objective posterior density [22]. By calculating the posterior probability density and calculating the probability function of the previous time, the prior probability density function is obtained, and the number of targets and their states at that time are obtained [16]. The implementation of PHD filtering needs to ensure the following assumptions [16].

For the tracked target, the motion of each target and the measurement are independent of each other.

In the system environment of target motion, the distribution of clutter should obey the Poisson distribution law and be independent of the measurement generated by the target. Implementations of PHD filter in robotics include [23-25].

The predicted multi-objective posterior probability is Poisson distribution [16]. Suppose $f\ (x)$ is a PDF of the target random set x in the state space X. $\delta(x)$ is a Dirac function with centre x, and its PHD is defined as $D(x)$ and PHD is expressed as

$$D(x) = E\left[\sum_{\zeta\in x} \delta_{\zeta}(x) f\right] = \int \sum_{\zeta\in x} \delta_{\zeta}(x) f(x) \delta(x). \tag{51}$$

In prediction process [16], if there is no derived target, the prediction equation of PHD filter is

$$D_{k|k-1}(x_i) = b(x_i) + \int_v f(x_i|\gamma)\, P_s(\gamma) \times D_{k-1|k-1}(\gamma) d\gamma, \tag{52}$$

Where $\mathrm{b(x}_i)$ represents the intensity function of the new target, $f(\mathrm{x}_i|\gamma)$ represents the probability density function of target state transition, $P_s(\gamma)$is the survival probability of the target.

In the update process [16], if the number of targets obeys Poisson distribution, the PHD filter update formula can be expressed as

$$D_{k|k}(x_i) = \left(1 - P_D(\gamma) + \sum_s^m \frac{P_D(x_i) f(Z_s|x_i)}{c(z_i)\mu_\lambda + \int_v f(x_i|\gamma) P_D(\gamma) \times D_{k|k-1}(\gamma) d\gamma}\right) D_{k|k-1}(x_i) \tag{53}$$

4.5 Discussion

PHD filtering has achieved no data association, but there are still some shortcomings:

(1) It is difficult to obtain the closed solution of PHD filter in general.

(2) In the process of multi-target tracking, it is difficult for the filter to distinguish short-range targets due to the mutual interference between targets and clutter, so that the result is not accuracy enough.

In view of the above, an improved PHD filtering algorithm has been developed, that is, the method of introducing trajectory label. The target state is expressed as $(x, y, x, y, L)^T$, and L is label [26]. The simulation results are as follows:

Table 1. Comparison of average error between improved algorithm and traditional algorithm under different detection rates [26]

P_D	Improved PHD filter	PHD filter
0.99	18.2356	38.3258
0.90	27.1266	42.2865
0.80	36.0776	46.1273
0.70	53.7403	51.6611

It can be seen from Table 1 that when the detection rate is high, the tracking accuracy of the improved algorithm is higher than that of the traditional algorithm; When the detection rate of the sensor is lower than a certain value, under the simulation conditions of this paper, when the detection rate PD = 0.70, the error of the improved algorithm is larger than that of the traditional algorithm [26].

Table 2. Comparison of average error between improved algorithm and traditional algorithm under different clutter density [26]

λ_c	Improved PHD filter	PHD filter
20	20.8639	42.3429
30	23.5252	46.2699
40	26.8369	55.5874
50	31.9430	76.5231

From the simulation data in Table 2, it can be seen that the traditional PHD algorithm is greatly affected by clutter, because the traditional algorithm will have more false targets in the high clutter environment; However, the improved algorithm still has high tracking accuracy in high clutter density environment [26].

JPDA algorithm also has some defects:

(1) JPDA algorithm can't deal with the disappearance of old targets and the appearance of new targets, so the number of targets in the tracking scene must be given.

(2) JPDA algorithm can't judge the start and end of the trajectory, so it must be handled separately.

(3) The method of updating the covariance matrix through all observations may aggravate the risk of false correlation, because the increased covariance matrix may introduce additional candidate observations in the correlation region.

5. Conclusion

Target tracking is widely used in civil and military applications. In this article, we summarize the related problems and solutions of target tracking. Target state estimation and multi-target data association are the core issues of target tracking. The data association method provided above is also a difficult problem to maintain target tracking immediately when the target is occluded and then reappears. The above-mentioned algorithms are the basis of the subsequent in-depth algorithm research. In practice, the corresponding algorithms need to be selected to deal with the corresponding problems, and sometimes the algorithms need to be combined with each other to better deal with the data association problems. In the future, researchers will conduct in-depth research on the algorithms and propose more effective and robust algorithms.

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